USING HMM IN STRATEGIC GAMES

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ABSTRACT

Informally, a game of incomplete information is a game where the players do not have common knowledge of the game being played. Among the aspects of the game that the players might not have common knowledge of are:

- payoffs
- Who the other players are
- What moves are possible
- How outcome depends on the action etc.

Incomplete information introduces additional strategic interactions and also raises questions related to "learning".

In this project, we analyze an approach to resolve a strategic game in which one of the players can assume two different types along the game. Our aim is to infer which type the opponent adopts at each period so that we can increase the player's odds of winning. To achieve that we use Hidden Markov model (HMM) to describe a concept of Hidden Markov Game with the inference algorithm by Baum-Welch. We discuss a hypothetical example of a repeated tennis game.

GAME THEORY TOOL KIT

Normal Form Strategic Game - Pure Nash Equilibria (Prisoner's Dilemma)

	confess	deny
confess	(-8,-8)	(0,-10)
deny	(-10,0)	(-1,-1)

Normal Strategic Game - Mixed Nash Equilibrium

	Rock	Paper	Scissor
Rock	(0,0)	(-1,1)	(1,-1)
Paper	(1,-1)	(0,0)	(-1,1)
Scissor	(-1,1)	(1,-1)	(0,0)

Incomplete Information Games-Bayesian Nash Equilibrium

Type 1 (with probability p)

	s1	s2
s1	(1,1)	(2,0)
s2	(0,2)	(3,3)

Type 2 (with probability 1-p)

	s1	s2
s1	(1,1)	(2,0)
s2	(0,2)	(0,0)

Markov Games : Repeated games of incomplete information

Type 1

	s1	s2
s1	(1,1)	(2,0)
s2	(0,2)	(3,3)

Type 2

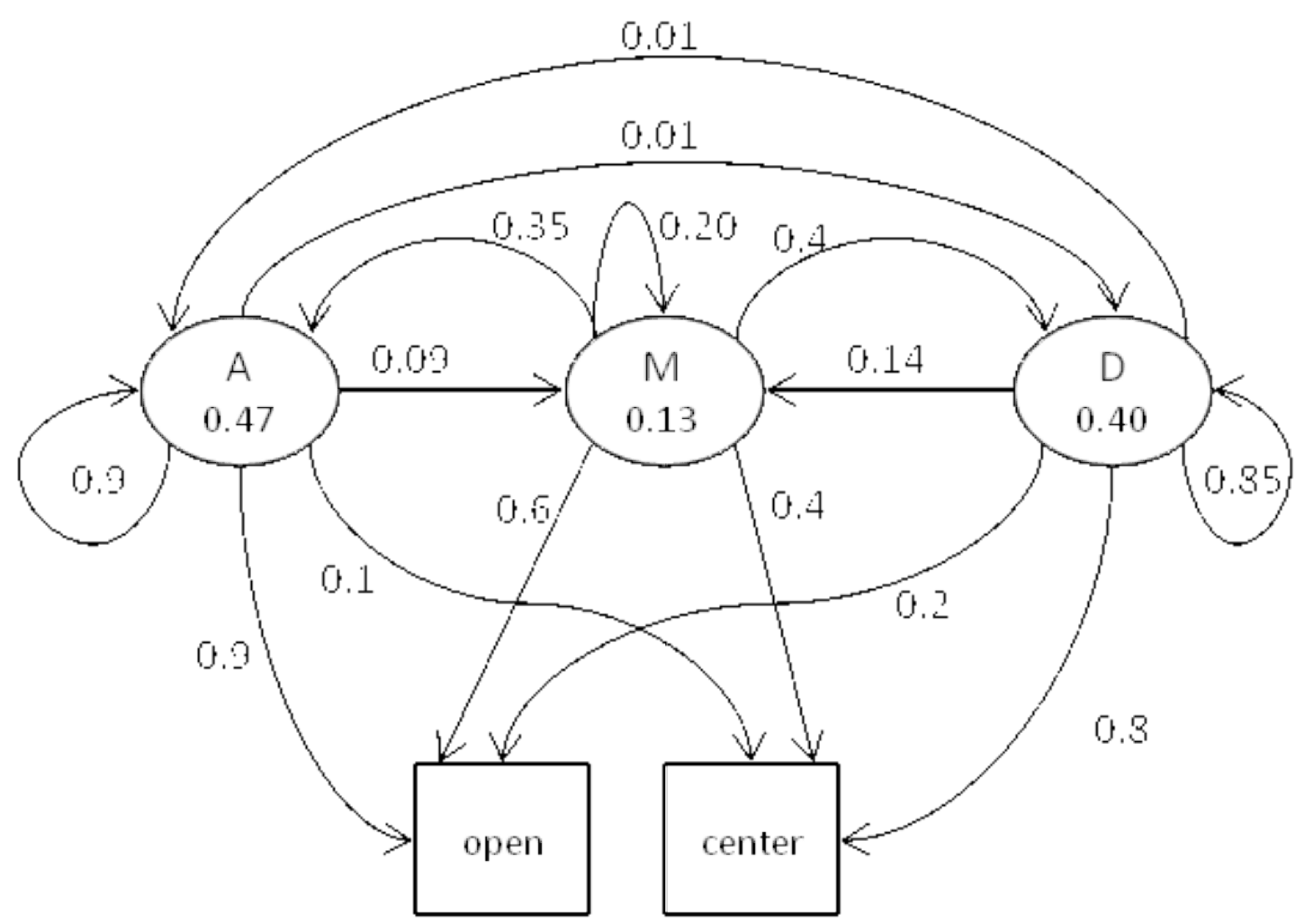
	s1	s2
s1	(1,1)	(2,0)
s2	(0,2)	(0,0)

State Transition Matrix

	Type 1	Type 2
Type 1	p	1-p
Type 2	q	1-q

HIDDEN MARKOV GAME

Hidden Markov Games : When state transition matrix is not common knowledge



PROPOSED SOLUTION

1. Represent the Hidden Markov Game as Hidden Markov Model (π, A, B)
2. Infer the matrix A
3. Solve the underline Markov Game and find the most probable type each opponent is playing
4. Play according to Mixed Nash Equilibrium.

INFERENCE METHOD - BAUM WELSH ALGORITHM

The joint probability;

$$p(Y, X|\pi, A, B) \quad = \left(p(x_1)p(y_1|x_1)\prod_{t=2}^{T} p(y_t|x_t)p(x_t|x_{t-1})\right)$$

Algorithm:

$$E-STEP: \; q(x) = p(x|y,\theta)$$
$$M-STEP: \; \theta^* = \arg\max_{\theta} p(y, x|\theta)$$

E-STEP:

$$C_\pi = \langle [s = x_1] \rangle_p$$
$$C_A = \sum_{t=2}^{T} \langle [r = y_t]\,[i = x_t] \rangle_p$$
$$C_B = \sum_{t=1}^{T} \langle [s = x_t]\,[j = x_{t-1}] \rangle_p$$

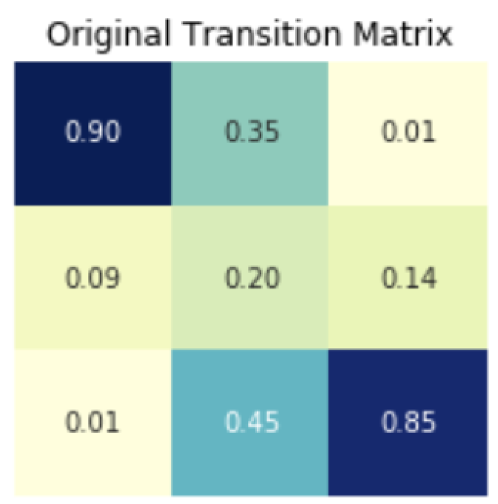
M-STEP:

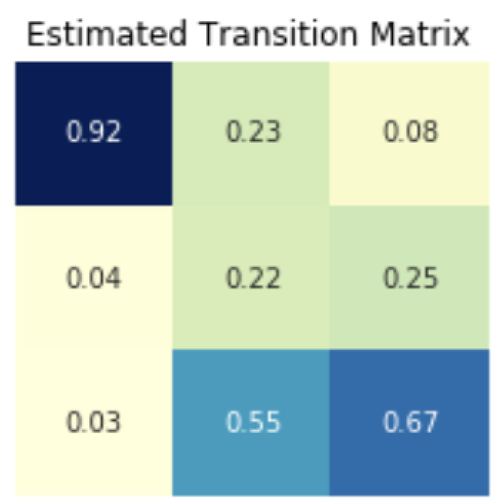
$$\pi_i = C_\pi(i)$$
$$A_{i,j} = \frac{C_A(i,j)}{\sum_i C_A(i,j)}$$
$$B_{k,i} = \frac{C_B(k,i)}{\sum_k C_B(k,i)}$$

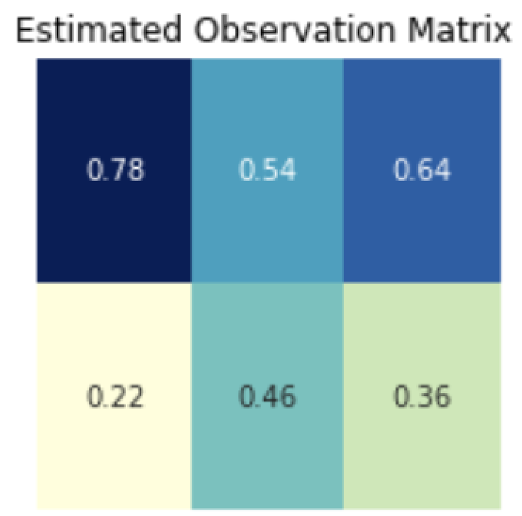
APPLICATION AND TEST

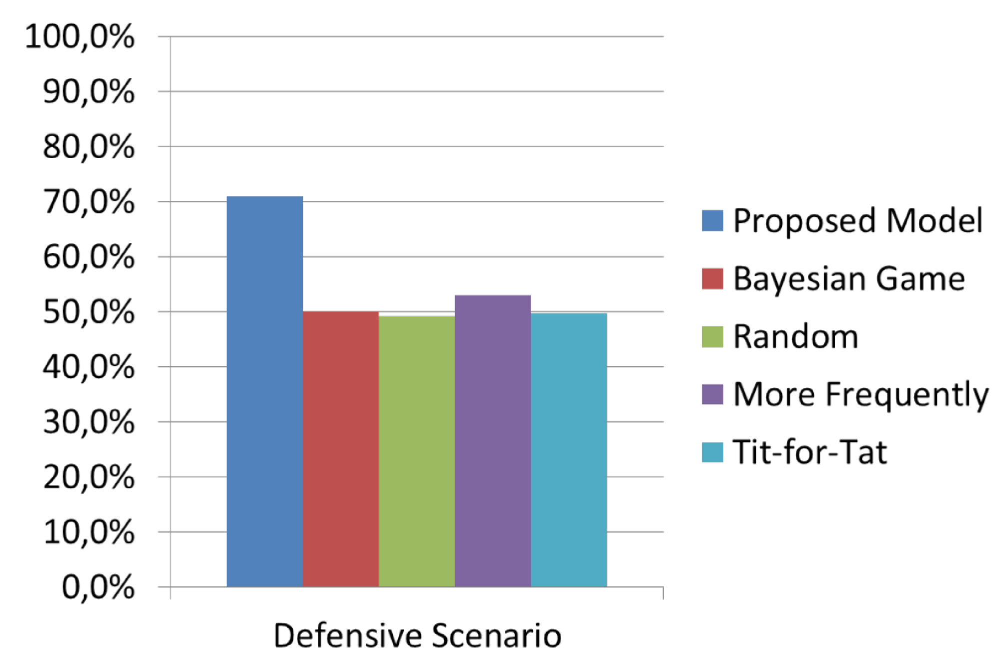
Payoff Matrices

	Aggressive		Moderate		Defensive	
	O	C	O	C	O	C
O	0.65, 0.35	0.89, 0.11	0.15, 0.85	0.80, 0.20	0.10, 0.90	0.55, 0.45
C	0.98, 0.22	0.15, 0.85	0.90, 0.10	0.15, 0.85	0.85, 0.15	0.05, 0.95









CONCLUSION

In conclusion, the initial transition matrix and the calculated observation matrix are fed to the constructed HMM model and the estimated transition matrix is obtained by using an EM inference method. Although the estimated transition matrix seems correct, according to the paper the proposed solution's hit rate appears to be less than random response hit rate . The reason for that unexpected result is probably due to the payoff matrices' values are actually some far from a real approach to the specified scenario at the paper.

REFERENCE

1. Mario Benevides, Isaque Lima, Rafael Nader, Pedro Rougemont, Using HMM in Strategic Games, 2014.